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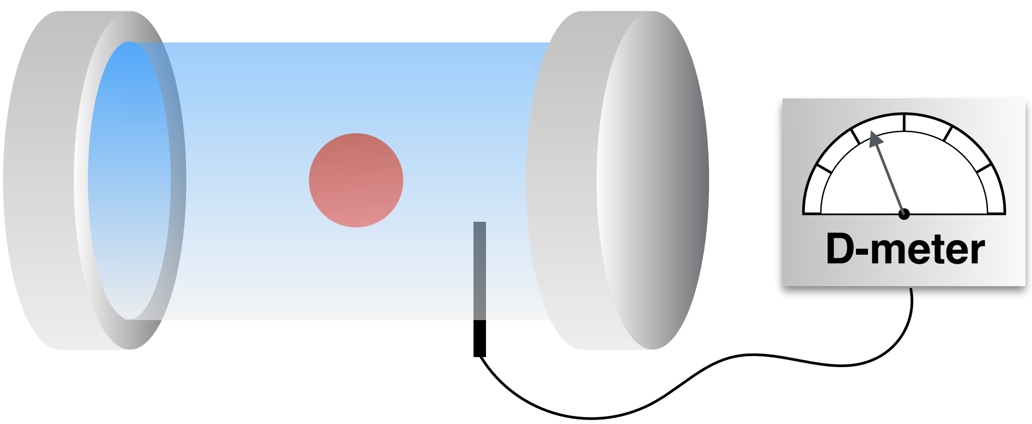
Probing the diamagnetic term in light-matter interaction

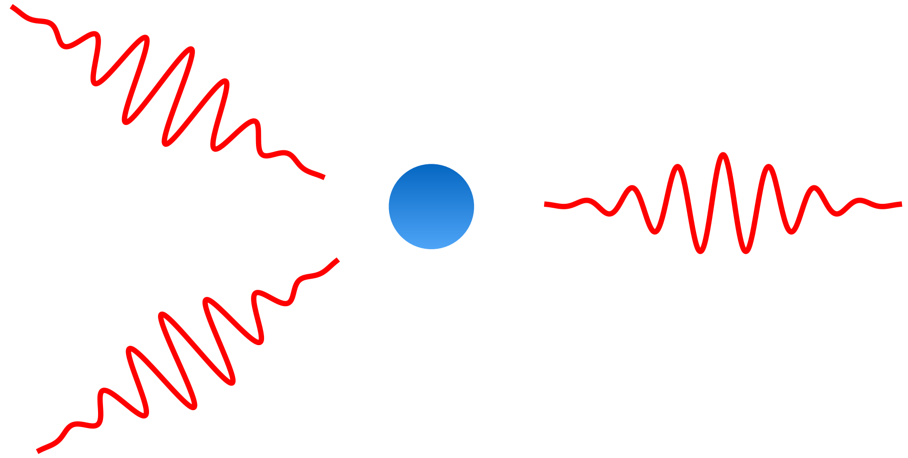


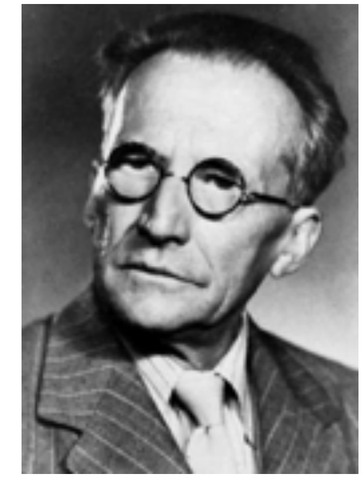
Tommaso Tufarelli
School of Mathematical Sciences
University of Nottingham

M. A. C. Rossi, M. Bina, M. G. A. Paris, M. G. Genoni, G. Adesso, T. Tufarelli

arXiv:1604.08506

Light-matter interaction in Quantum Mechanics





Minimal Coupling Hamiltonian

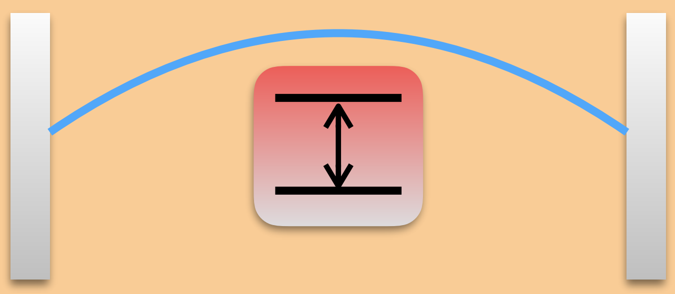
$$\mathcal{H} = \sum_j \underbrace{\frac{(\hat{\mathbf{p}}_j - Q_j \hat{\mathbf{A}})^2}{2m_j}}_{\text{kinetic}} + \underbrace{V(\{\hat{\mathbf{x}}_j\})}_{\text{electrostatic}} + \underbrace{\mathcal{H}_{EM}}_{\text{radiation}}$$

 <u>Accurate</u> (for non-relativistic charges)

 Computationally <u>intractable</u>

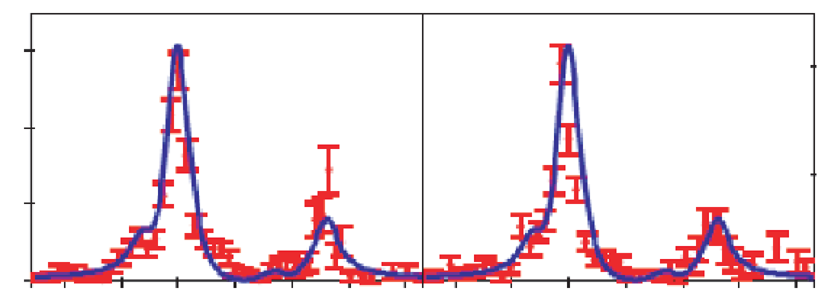
Approximate models are needed!

few degrees of freedom



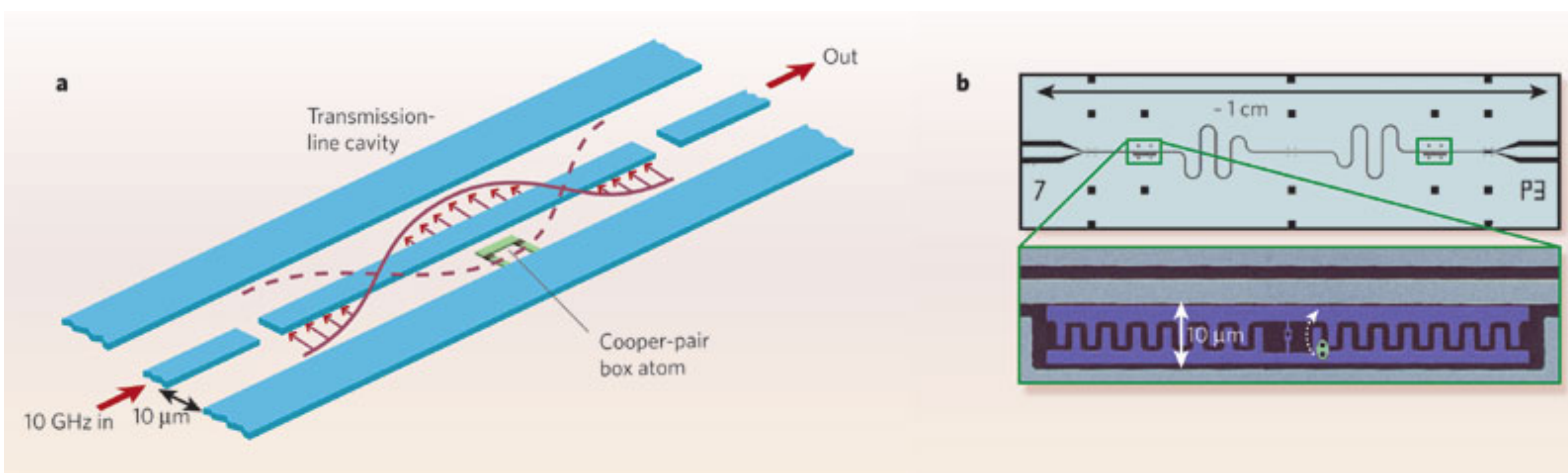
E.g. single mode field + 2-level atom

understanding fundamental processes



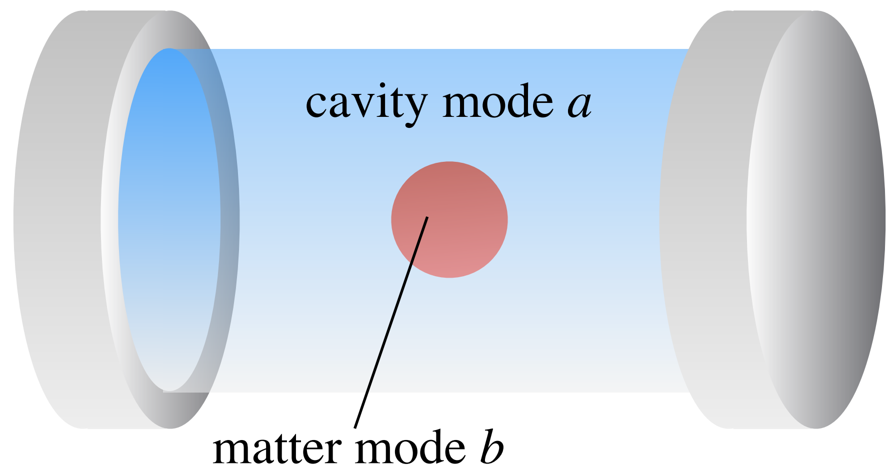
A.Boca et al, PRL. **93**, 233603 (2004)

Technologically relevant (e.g. Circuit QED)



R. J. Schoelkopf & S. M. Girvin Nature **451**, 664 (2008)

Dicke model



$$[\hat{a}, \hat{a}^\dagger] = 1$$

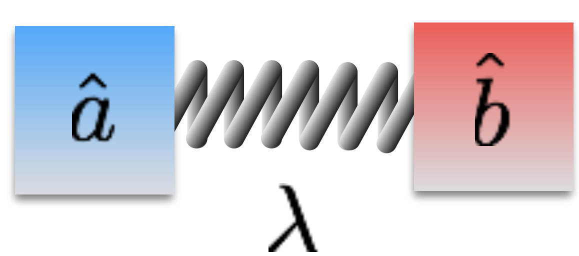
light mode

$$[\hat{b}, \hat{b}^\dagger] = 1$$

matter mode

$$\mathcal{H} = \underbrace{\omega_a \hat{a}^\dagger \hat{a}}_{\mathcal{H}_{EM}} + \underbrace{\omega_b \hat{b}^\dagger \hat{b}}_{\frac{\hat{p}^2}{2m} + \hat{V}} + \underbrace{\lambda(\hat{b} + \hat{b}^\dagger)(\hat{a} + \hat{a}^\dagger)}_{\hat{\mathbf{p}} \cdot \hat{\mathbf{A}}} + D\underbrace{(\hat{a} + \hat{a}^\dagger)^2}_{\hat{A}^2}$$

Analogous* to coupled oscillators:



(with a couple more springs)

A^2 term `controversy' in a nutshell

$$\mathcal{H} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \lambda(\hat{b} + \hat{b}^\dagger)(\hat{a} + \hat{a}^\dagger) + D(\hat{a} + \hat{a}^\dagger)^2$$

Suppose to know* these parameters

expected $D \propto \lambda^2$

Q: what is the most appropriate value of *D*?

Relevant for the **Ultrastrong Coupling Regime**

$$\lambda \sim \mathcal{O}(\omega_a) \sim \mathcal{O}(\omega_b)$$

$D \leq \frac{\lambda^2}{\omega_b} - \frac{\omega_a}{4}$ enables the **Dicke Phase Transition**

Excerpts from the A² term debate (1)

ARTICLE

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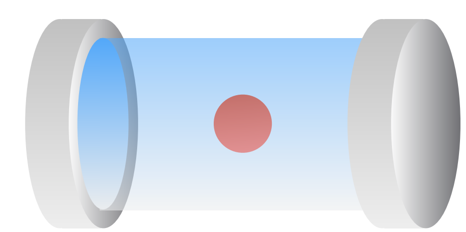
No-go theorem for superradiant quantum phase transitions in cavity QED and counter-example in circuit QED

Pierre Nataf[1] & Cristiano Ciuti[1]

In cavity quantum electrodynamics (QED), the interaction between an atomic transition and the cavity field is measured by the vacuum Rabi frequency Ω_0. The analogous term 'circuit QED' has been introduced for Josephson junctions, because superconducting circuits behave as artificial atoms coupled to the bosonic field of a resonator. In the regime with Ω_0 comparable with the two-level transition frequency, 'superradiant' quantum phase transitions for the cavity vacuum have been predicted, for example, within the Dicke model. In this study, we prove that if the time-independent light-matter Hamiltonian is considered, a superradiant quantum critical point is forbidden for electric dipole atomic transitions because of the oscillator strength sum rule. In circuit QED, the analogous of the electric dipole coupling is the capacitive coupling, and such no-go property can be circumvented by Cooper pair boxes capacitively coupled to a resonator, because of their peculiar Hilbert space topology and a violation of the corresponding sum rule.

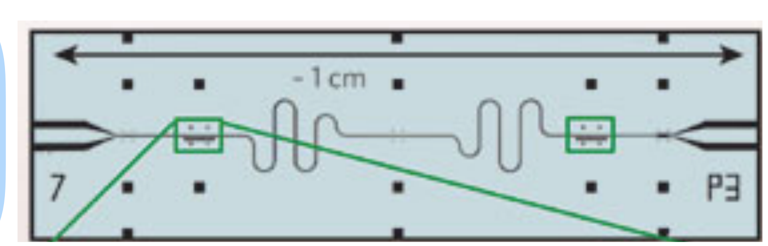
$$D \geq \frac{\lambda^2}{\omega_b}$$

for Cavity QED



$$D = 0$$

for Circuit QED



Excerpts from the A^2 term debate (2)

PRL **107,** 113602 (2011) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2011

Superradiant Phase Transitions and the Standard Description of Circuit QED

Oliver Viehmann,[1] Jan von Delft,[1] and Florian Marquardt[2]

[1]*Physics Department, Arnold Sommerfeld Center for Theoretical Physics, and Center for NanoScience, Ludwig-Maximilians-Universität, Theresienstraße 37, 80333 München, Germany*

[2]*Institut for Theoretical Physics, Universität Erlangen-Nürnberg, Staudtstraße 7, 91058 Erlangen, Germany*

(Received 29 March 2011; published 8 September 2011)

We investigate the equilibrium behavior of a superconducting circuit QED system containing a large number of artificial atoms. It is shown that the currently accepted standard description of circuit QED via an effective model fails in an important aspect: it predicts the possibility of a superradiant phase transition, even though a full microscopic treatment reveals that a no-go theorem for such phase transitions known from cavity QED applies to circuit QED systems as well. We generalize the no-go theorem to the case of (artificial) atoms with many energy levels and thus make it more applicable for realistic cavity or circuit QED systems.

$$D \geq \frac{\lambda^2}{\omega_b}$$

for circuit QED

Excerpts from the A² term debate (3)

PRL **112,** 073601 (2014) PHYSICAL REVIEW LETTERS week ending 21 FEBRUARY 2014

Elimination of the *A*-Square Problem from Cavity QED

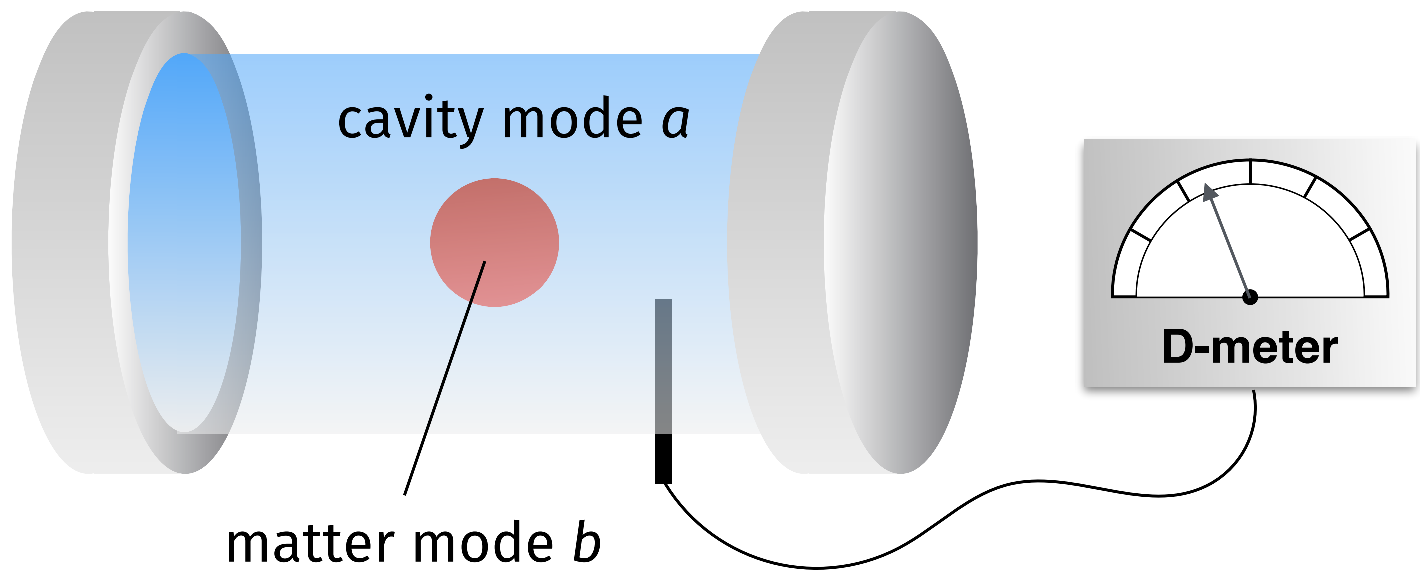
$$D = 0$$

for Cavity QED!

?

Our proposal

Estimate *D* with an experiment!



… because the theory is too difficult!

Research question (1)

$$\mathcal{H} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \lambda(\hat{b} + \hat{b}^\dagger)(\hat{a} + \hat{a}^\dagger) + D(\hat{a} + \hat{a}^\dagger)^2$$

How much information about D is contained in the ground state?

$$|G\rangle \not\simeq |0\rangle_a \otimes |0\rangle_b$$

Quantum Cramer-Rao bound

Estimator $\hat{D}$, N measurements $\langle \hat{D} \rangle = D_{\text{est}}$

$$\delta D_{\text{est}}^2 \geq \frac{1}{N H(D)}$$

variance of estimated D

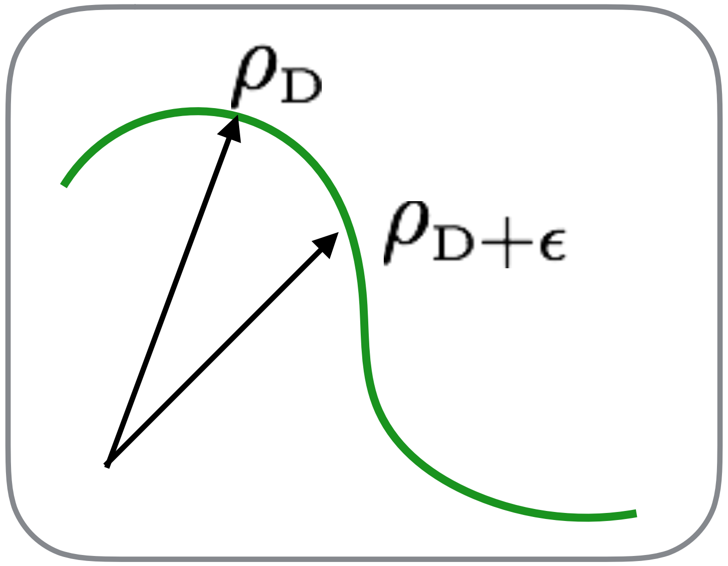
Quantum Fisher info

Quantum Fisher Information (QFI)

$$H(D) = \mathrm{Tr}(\rho L^2)$$

$$\rho = |G\rangle\langle G| \qquad\qquad 2\frac{\partial}{\partial D}\rho = L\rho + \rho L$$

(ignore if unfamiliar with notation)



Experimentally relevant regime

$$\lambda \lesssim 10\% - 20\%\, \omega_{a,b}$$

$$D \lesssim \lambda^2/\omega_b$$

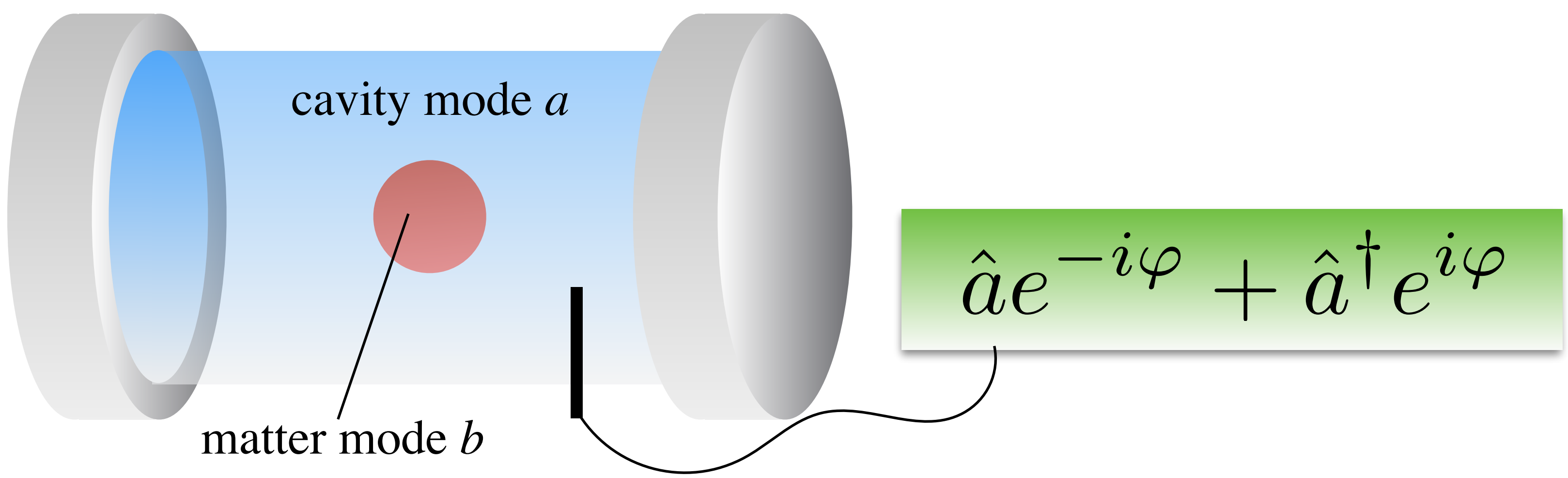
$$H(D) = \frac{2}{(4D + \omega_a)^2} + \mathcal{O}(\lambda^2)$$

"error bar": $\delta D_{\text{est}} \sim \frac{\omega_a}{\sqrt{N}}$

$H(D)$ = fundamental, *measurement-independent* quantity

How much information can we access via feasible measurements?

Homodyne detection



Cramer-Rao bound

$$\delta D^2_{\text{est}} \geq \frac{1}{F(D)}$$

(for *N* measurements)

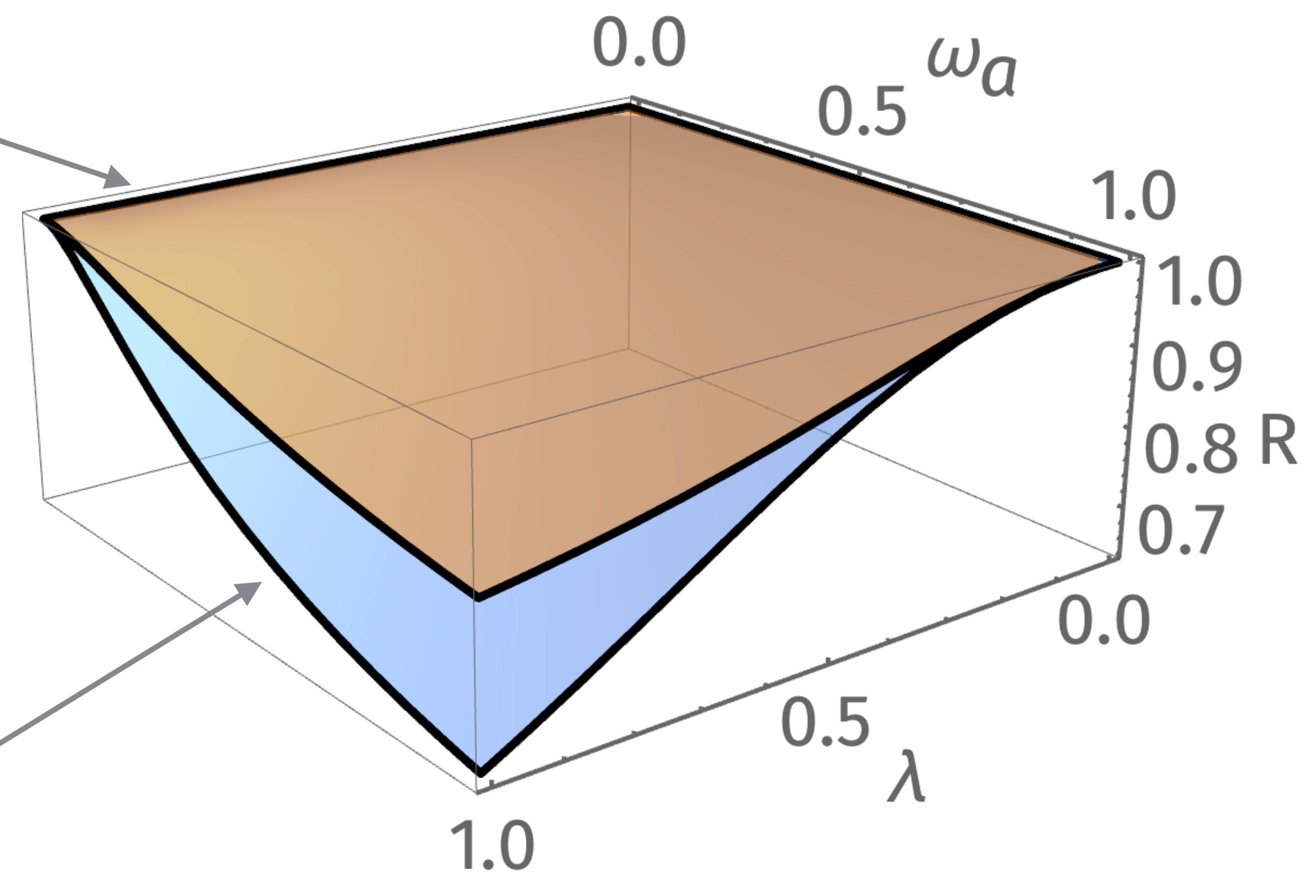
Classical Fisher info

$$F(D) = \int dx [\partial_D p(x|D)]^2 / p(x|D)$$

Near-optimality of Homodyne detection

$$R_a = \frac{F(D)}{H_a(D)}$$

$H_a(D)$ = QFI of reduced field state



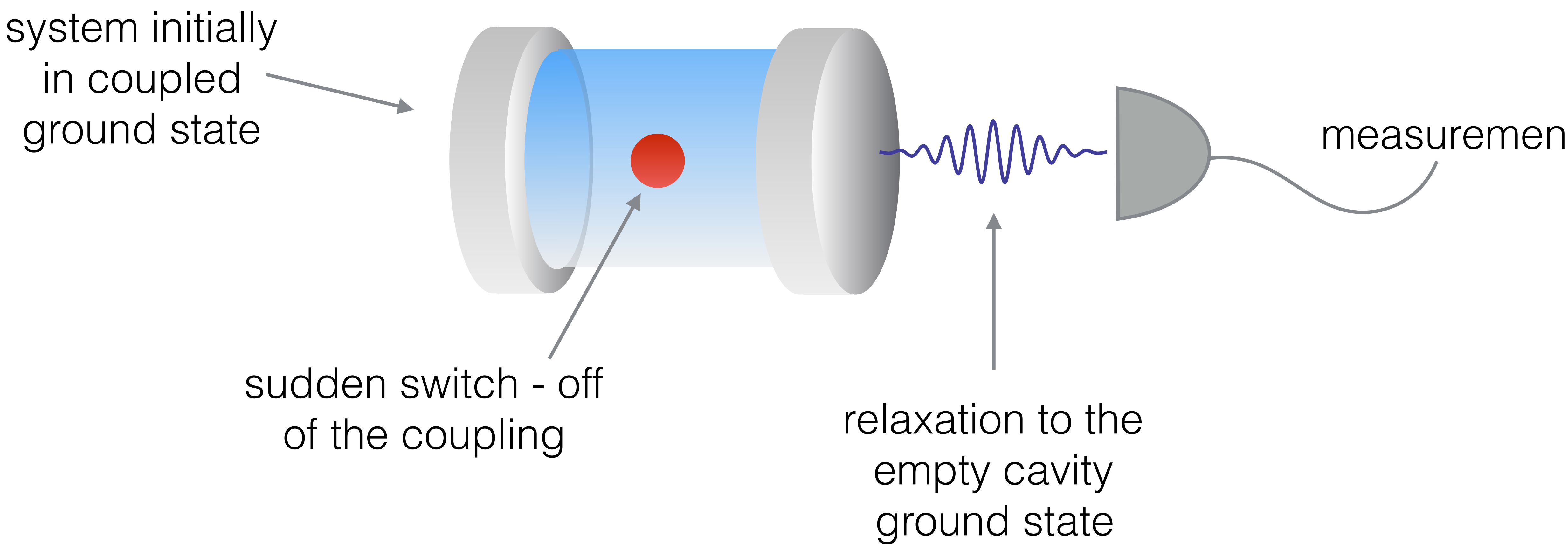
$$R = \frac{F(D)}{H(D)}$$

Small / moderate coupling

$$R \simeq 1 - \frac{8\omega_a^2}{(\omega_a + \omega_b)^4}\lambda^2 + O(\lambda^4)$$

How to access the cavity field?

Non adiabatic switch-off of light matter interactions (λ, D)



Experimentally feasible!

nature Vol 458 | 12 March 2009 | doi:10.1038/nature07838

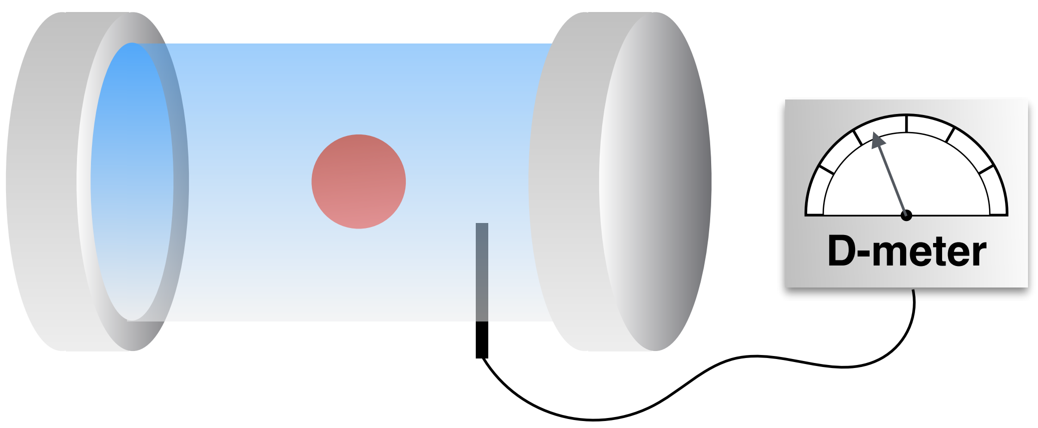
LETTERS

Sub-cycle switch-on of ultrastrong light–matter interaction

Conclusions

Our main message:

The A^2 term debate could be settled experimentally



We quantified how much information on D is encoded in the ground state of the Dicke model (via Quantum Fisher Information)

Homodyne detection is nearly optimal in relevant parameter regimes

The cavity field can be accessed via non-adiabatic switch-off of interactions

Thanks for your attention!

M. A. C. Rossi, M. Bina, M. G. A. Paris, M. G. Genoni, G. Adesso, T. Tufarelli

arXiv:1604.08506

a bit more
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